Calculus on the TI-85/TI-86

Tolerance

Since the calculator can't do derivatives or integrals symbolically, it has to do them numerically. Thus, it can only approximate the numerical value of derivatives and definite integrals. The variables TOL and δ are used to set the accuracy of these values. They must be set to positive numbers.

Let's set TOL=10^{-5} and δ=0.001 so that the following results will be the same as yours.

1. Enter the tolerance editor.
   TI-85 | Press **2nd TOLER**.
   TI-86 | Press **2nd MEM F4:TOL**.
2. Enter the values for TOL and δ.
   Type **10 ^ (-) 5 ENTER .001 ENTER**.
3. Exit the editor.
   Press **EXIT**.

Derivatives

In the Calc(ulus) menu there are 3 derivative functions.

nDer :Basic numerical first derivative using the symmetric difference quotient
[f(x-δ) - f(x+δ)]/(2δ) :Affected by δ.
der1 :More advanced first derivative. :Not affected by TOL or δ.
der2 :More advanced second derivative. :Not affected by TOL or δ.

The syntax for all 3 are

***function*(*expression*,*variable*,*value*)** where

- ***function*** :one of the 3 functions above.
- ***expression*** :the expression of the function you want to take the derivative of.
- ***variable*** :the variable to take the derivative with respect to.
- ***value*** :the value of the variable to evaluate the derivative at. This is optional if the variable is already defined as a real number.

Example#1

nDer(x^3,x,5) will return the derivative of x^3 evaluated at 5. In other words, if $y=x^3$ then $dy/dx = 3x^2$. Which at x=5, $dy/dx = 3(5)^2 = 75$. Try it yourself, if you aren't already in the CALC menu type **2nd CALC**. Then press **F2:nDer x-VAR ^ 3 , x-VAR , 5) ENTER**. The calculator will return "75.000001". Why didn't you get 75 exactly? Try der1(x^3,x,5). Does it do better?

Example#2: For f(x)=|x| evaluate f'(0).

0. Since the difference quotient , [f(x+h)-f(x)]/h, is 1 when h>0 and -1 when h<0, f'(x) is undefined at x=0.
1. Try nDer(abs(x),x,0) it will return "0". Why?
2. Try der1(abs(x),x,0) it will return an error.

Thus from the above examples, when evaluating the derivative of a function at a specified value of x, der1 is the better choice.

Example#3: For y=sin(t), find y" at t = π/3.

0. By hand, y'=cos(t) , y"=-sin(t) and $-\sin(\pi/3)=-\sqrt{3}/2=-.866025403784$.
1. der2(sin(x),x,π/3) returns "-.866025403784".
2. This can also be done as nDer(der1(sin(x),x),x,π/3) which returns "-.86602525945".
3. While nDer(nDer(sin(x),x),x,π/3) returns "-.8660251175".
4. And, der1(nDer(sin(x),x),x,π/3) and der1(der1(sin(x),x),x,π/3) will return errors.

The above example shows the main limitation of der1 and der2. In that, they can not be used to take the derivative of certain functions. In this case, ones dealing with the above derivative functions.

Graphs and Derivatives

Example#4: Graph the line tangent to $y=x^2$ at x=2 on the calculator.

0. It can be shown by hand that for x=2, y=4 and that y=4x-4 is tangent to the curve $y=x^2$ at the point (2,4).
1. Graph $y=x^2$ in a window containing the point (2,4), i.e. [-10,10]by[-10,10].
2. Enter the tangent line function by typing
   TI-85 | **MORE F1:MATH MORE MORE F3:TANLN**
   TI-86 | **MORE F1:MATH MORE MORE F1:TANLN**
3. Move the cursor as close to the point (2,4) as you can get and enter the point.
   TI-85 | press **(cursor right)** until you get to the point (2.063492...,4.257999...) then press **ENTER**.
   TI-86 | type **2 ENTER**
4. The calculator will graph the line tangent to the point entered and print dy/dx at the bottom of the screen.
   The TI-85 will return "dy/dx=4.126984127".
   The TI-86 will return "dy/dx=4".

To get the TI-85 to return the correct answer of dy/dx=4, the window must be set a certain way. For example, the window [-12.6,12.6]by[-6.2,6.2] would allow you to get to the point (2,4) exactly. See next example for another way.

In the following examples, GRAPH is at the beginning of a list of keystrokes solely to make sure that everyone is at the same place. Also, the window settings are chosen for the TI-85 users. TI-86 users can set the window to anything as long as the key points are shown.

Example#5: Graph the line tangent to $y=e^x$ at x=1. (Alt. method)

1. Enter $y1=e^x$.
2. Since the point (1,e) is in the window [-6.3,6.3]by[-3.1,3.1], we can use the ZDECM function. Type **GRAPH F3:ZOOM MORE F4:ZDECM** and wait for the graph. The window is now set such that the cursor moves by 0.1 units horizontally and vertically.
3. Enter the tangent line function by typing
   TI-85 | **GRAPH MORE F1:MATH MORE MORE F3:TANLN**
   TI-86 | **GRAPH MORE F1:MATH MORE MORE F1:TANLN**
4. Move the cursor to the point (1,e) and enter the point.
   Press **(cursor right)** until you get to the point (1,2.71828...) then press **ENTER**.
   (On the TI-86 you can still type **1 ENTER** to go directly to the point)
5. The calculator will graph the line tangent to the point entered and print "dy/dx=2.7182818285" at the bottom of the screen.

Example#6: Use the dy/dx function to approximate y' at x=5 where y=Log(x).

1. Check the dy/dx's mode.
   Press **2nd MODE**.
   The last entry sets whether the dy/dx function uses the der1 command (dxDer1) or the nDer command (dxNDer). As we have seen before, der1 is usually the better choice. Thus, this option should be set to dxDer1.
2. Graph y=Log(x) in the ZDECM window [-6.3,6.3]by[-3.1,3.1].
3. Enter the dy/dx function by typing
   TI-85 | **GRAPH MORE F1:MATH F4:dy/dx**
   TI-86 | **GRAPH MORE F1:MATH F2:dy/dx**
4. Move the cursor to the point (5,Log(5)) and enter the point.
   Press **(cursor right)** until you get to the point (5,.69897...) then press **ENTER**.
   (On the TI-86 you can still type **5 ENTER** to go directly to the point).
5. The calculator prints "dy/dx=.08685889638".

In the above three examples, dy/dx is stored into the ANS variable for access from the command line.

Example#7: Using the calculator to approximate the inflection point of $y=x^3+2x^2-3x+5$.

1. Graph $y=x^3+2x^2-3x+5$ in the standard window.
2. Enter the inflection point function by typing
   TI-85 | **GRAPH MORE F1:MATH MORE F3:INFLC**
   TI-86 | **GRAPH MORE F1:MATH MORE F1:INFLC**
3. TI-86 ONLY: Enter the left(lower) and right(upper) bounds.
   Press **(cursor left)** until the cursor is to the left of the I.P. and press **ENTER**.
   Press **(cursor right)** until the cursor is to the right of the I.P. and press **ENTER**.
4. Move the cursor close to the I.P. with **(cursor left)** and **(cursor right)**. Press **ENTER**.
5. The calculator returns "x=-.6666666667 y=7.5925925926" as the I.P.
6. Since both x and y repeat, they should be rational. So lets get the calculator to convert y to a fraction. You should recognize x as -2/3 .
   Return to the command line by pressing **EXIT EXIT EXIT**.
   Both x and y are stored in the variables of the same name. Thus, if we type **2nd alpha y 2nd MATH F5:MISC MORE F1:>Frac ENTER**. The calculator returns "205/27".
7. Thus, the I.P.=(-2/3 , 205/27).

Note: Step #6 will only work if you haven't done anything to change the value of y in the calculator.

Newton's Method

Example#8: Apply Newton's method to solve $\sin(x)e^x=3$ to the hundredths with initial guess of $x_0=5$.

1. Bring all non-zero terms to one side.
   $\sin(x)e^x-3=0$
2. Set x to x_0 and the non-zero side of the equation in y1.
   Type **2nd alpha y 1 ALPHA = SIN x-VAR X 2nd e^x x-VAR - 3 ENTER** to store y1.
   Type **5 STO► x-VAR ENTER** to store x.
3. Execute Newton's method and store back into x.
   Type **x-VAR - 2nd alpha y 1 ÷ 2nd CALC F3:der1 2nd alpha y 1 , x-VAR) STO► x-VAR**.
   The calculator returns "3.57992283293".
4. Press **ENTER** repeatedly to reexecute the last command without having to retype it until desired accuracy is achieved.
   In this case, after pressing **ENTER** 4 times the results all round to 3.14 .
   Press **ENTER** a couple of more times and the number stops changing.

Note: We use y1 solely to save typing. Sometimes, it's easier to find y' by hand and not use y1. You can also use the GRAPH menu's function editor, y(x)=, to enter and delete y1.

Definite Integrals

The calculators have only one numerical integration function, fnInt, which is affected by TOL. It's syntax is

fnInt(*expression*,*variable*,*lower limit*,*upper limit*) ,where

expression :the expression of the function you want to take the integral of.
variable :the variable to integrate with respect to.
lower limit :the lower limit of integration, a.
upper limit :the lower limit of integration, b.

Example#9: Evaluate $\int_2^7 (x^5-3)\,dx$.

0. By hand, $\int_2^7 (x^5-3)\,dx = [\ x^6/6 - 3x\ ]_2^7 = 7^6/6 - 3*7 - (\ 2^6/6 - 3*2\ ) = 19582.5$.
1. Enter fnInt(x^5-3,x,2,7) by typing **2nd CALC F5:fnInt x-VAR ^ 5 - 3 , x-VAR , 2 , 7) ENTER**. It will return "19582.5".

Example#10: Find the integral from 0 to 1 of $e(-x^2)$ with respect to x.

0. We can't do this one exactly by hand since this function doesn't have an anti-derivative. At least, there's not one that doesn't involve an infinite series.
1. Enter fnInt(e^(-x^2),x,0,1). It will return ".746824132782". But how accurate is it? TOL is 0.00001. Thus, the exact answer is between 0.746814132782 and 0.746834132782 (Note the 5th decimal places).
2. Let's see just how accurate it is? Change TOL to 10^{-13}. Since, the calculator will only display 12 digits, this will make the calculator as accurate as it can get.
   Execute the command from step 2 again. You should be able to do this by just pressing **ENTER**. The calculator will return ".746824132812". Thus the first answer was actually accurate to the 7th decimal place.

Thus, why not just leave TOL=10^{-13}? Did you notice how much longer it took for the calculator to evaluate the integral the second time? As a rule, the lower TOL is, the longer the wait. Also, since the calculator only shows 12 significant digits, setting TOL < 10^{-13} rarely improves accuracy.

Example#11: Find the integral from 0 to $6\pi/13$ of $10^{(-x^2)}\tan(x)$ with respect to x rounded to the 3rd decimal place in the least amount of time.

1. To be able to round accurately to the 3rd decimal place, TOL must equal 0.0001 or less so that the error would start at the 4th decimal place.
2. Set TOL=0.0001 and enter fnInt(10^(-x^2)tan(x),x,0,6π/13). It returns ".265241018902". Thus, the answer is 0.265 .
3. Actually this method has a 10% chance of rounding to the wrong number. Why?

Graphs and Integrals

Example#12: Use the ∫f(x) function to approximate $\int_1^{2.3}(x^3-4x)dx$.

1. Graph $y=x^3-4x$ in the ZDECM window [-6.3,6.3]by[-3.1,3.1].
2. Enter the ∫f(x) function by typing
   - TI-85 | **GRAPH MORE F1:MATH F5:∫f(x)**
   - TI-86 | **GRAPH MORE F1:MATH F3:∫f(x)**
3. Enter the lower limit of integration.
   - TI-85 | press **(cursor right)** until you get to the point(1,-3) then press **ENTER**.
   - TI-86 | type **1 ENTER**
4. Enter the upper limit of integration.
   - TI-85 | press **(cursor right)** until you get to the point(2.3,2.967) then press **ENTER**.
   - TI-86 | type **2.3 ENTER**
5. The TI-86 will shade in the related regions.
6. Both calculators will return "∫f(x)=-1.833975". This result is now stored is ANS.

Even though the calculator drops off the "dx" from its integral function, ∫f(x), it's still a required part of integral notation.

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