Advanced Functions on the TI-89/92

Polynomial Root Finder: (Flash Application that might have to be loaded onto older TI89/92+'s)
The polynomial solver can be used to solve polynomial equations of degree 2 or higher.

Example #1: Solve $x(x^2 - 3x) = 4$

1. Put the equation in standard form.
   $x^3 - 3x^2 - 4 = 0$
2. Enter the polynomial solver.
   Type **APPS 1:FlashApps "Polynomial Root Finder" "New..."**.
3. Enter the degree(order) of the polynomial.
   Type **3 ENTER**.
4. Enter the coefficients starting with the leading coefficient.
   Type **1 ENTER (-) 3 ENTER 0 ENTER (-) 4**. Note: you have to type in 0 for the missing x term's coefficient.
5. Press **F5:SOLVE** and wait.
6. Read the solutions. Note, depending on the mode of the calculator, imaginary solutions will be displayed as "Non-Real" or the form of either rectangular:a+bi or polar:e^(θi)*r.

   For rectangular mode, it returns

   x_1= 3.35530139761

   x_2= -.177650698804+1.07730381285i

   x_3= -.177650698804-1.07730381285i

   To see the rest of the second and third solutions, put the cursor on the solution and press **(cursor right)** to scroll through that solution.
   Thus, rounded to the 4th decimal place, the solutions are
   3.3553
   -0.1777 + 1.0773i
   -0.1777 - 1.0773i

Note: This function is prone to approximation errors, i.e. 0's quite often show up as numbers like 3E-12=0.000000000003 . So if you get an answer like 1.9999999324, plug 2 in for x by hand to see if the solution is 2 or 1.9999999324 . For example, try solving $x^5+5x^4+10x^3+10x^2+5x+1=0$ which has only one solution of x=-1 with multiplicity of 5.

Simultaneous Eqn Solver (Flash Application that might have to be loaded onto older TI89/92+'s)
The Simultaneous equations solver is used to solve linear systems of square dimensions 2 by 2 or higher.

Example#2 :Solve 3x - 4y = 13
x + 6y = 8

1. Put all equations into general form, i.e. Ax + By = C.
2. Enter the Simultaneous equation solver.
   Press **APPS 1:FlashApps "Simultaneous Eqn Solver" "New..."**.
3. Enter the number of equations or variables.
   In this case type **2 ENTER ENTER**.
4. Enter the coefficients, in order, and the result for 1st equation.
   Type **3 ENTER (-) 4 ENTER 13 ENTER**.
5. Repeat step #4 for the rest of the equations.
   Type **1 ENTER 6 ENTER 8 ENTER**.
6. Press **F5:SOLVE**.
7. Read the answer.
   It returns x_1=5, and x_2=1/2 which means that x=5 and y=1/2. Thus the solution set is {(5,1/2)} .
8. You can press **F4:Coef** if you want to edit the coefficients.

Note: If an @ appears in the answer, such as x_1=3-2@1 and x_2=@1, then there are an infinite number of solutions.

TABLES

The table feature can be used to see a table of values for the function(s) in y1, and/or y2, etc.

Example#3 :Demonstrate that $(1 + 1/x)^x$ approaches e=2.71828182845... as x approaches ∞.

1. Enter the function into y1.
   Press ♦ **Y= (1 + 1 ÷ x) ^ x ENTER**.
2. Enter the table menu.
   Press ♦ **TABLE**.
3. Enter the table setup screen.
   Type either **F2:TBLST** or ♦ **TBLST**.
4. Set tblStart to the value of x that you what the table to start at.
   Set this to 1 by typing **1 (cursor down)**.
5. Set Δtbl to how much you want x to change between rows of the table.
   Set this to 1 by typing **1 ENTER**.
6. Leave Indpnt to AUTO and Graph<->Table to OFF.
7. Return to the table screen.
   Press **ENTER**.
8. Examine the table using (cursor up) and (cursor down).
   Well the numbers under y1 seem to be heading toward 2.7182818... , but they aren't really near it yet.
9. Experiment with the table setup.
   Let's try Δtbl=1,000. So type **F2:TBLST 1 (cursor down) 1000 ENTER ENTER**.
   It looks better. At least it is accurate to the 3rd decimal place.
   Let's try ΔTbl=1,000,000. So type **F2:TBLST 1 (cursor down) 10 ^ 6 ENTER ENTER**..
   Much better. Now it's showing the same number for x>1, but we can see pass the 4th decimal place.
10. Change the width of the cells.
    Type **F1:Tools 9:Format (cursor right)**. Then press **(cursor down)** until 12 is highlighted. Type **ENTER ENTER**.
    Now we can see it's accurate to the 4th decimal place.
    From this we can conclude that $(1+1/x)^x$ seems to approach e as x approaches ∞. But note that this is not proof that $(1+1/x)^x$ approaches e, see problem in next section. A formal proof requires calculus. But, this is enough for you to take my word that it does approach e.

When Good Calculators Go Bad

I came across this problem when working with the example used in the table section above (Example#3 :Demonstrate that $(1 + 1/x)^x$ approaches e=2.71828182845... as x approaches ∞). We'll examine this both with tables and with graphs.

With Tables:

1. If it's not still there, set $y1=(1+1/x)^x$. You also need the cell width set to 12.
2. Set Δtbl=10,000,000 and go to the table.
   Does anything look a little peculiar? Look at when x=30000001.
3. Set Δtbl=3,000,000,000 and go to the table. Note that you can use the EE key to type in Δtbl a bit quicker.
   Does it still look like it's converging to e=2.7182818285...?
4. Set $\Delta tbl=1x10^{13}$ and go to the table.
   How about now? Are you questioning the conclusion we made in example#3?

With Graphs:

1. If not already done, set $y1=(1+1/x)^x$.
2. Set the window to [0,10] by [0,8] and graph.
   Does it look like it could have a horizontal asymptote at y=e=2.7182818285...?
3. Set the window to [0,1000000] by [0,8] and go into trace mode.
   Does it look like the function is approaching 2.7182818285...?
4. Set the window to $[0,1x10^{12}]$ by [0,8].
   How about now? Does it still look like it has a horizontal asymptote?
5. Try $[0,1x10^{13}]$ by [0,8].
   Any ideas yet?
6. Finally, try $[0,5x10^{13}]$ by [0,8] and this time set xscl to $1x10^{13}$.
   What happened at $2x10^{13}$?

The Limit of the Calculator.

First a couple of definitions. By a small number I mean a number close to 0, and by a large number I mean a number far from 0. Thus, 0.00000000005 is smaller than -5,000,000,000 even though 0.00000000005 is greater than -5,000,000,000.

The calculator can deal with numbers as small as $1\text{x}10^{-999}$ and as large as $9.9999999999999\text{x}10^{999}$ with no problem as long as it doesn't try to mix large, small or average sized numbers at the same time. Then, there can be a problem depending on what you try to do with them. Multiplication is no problem, but addition is another story.

The next thing to understand is that the calculator stores at most 14 consecutive digits of any number. It displays the 1st 12 of those digits and keeps the other 2 hidden. To see this. On the command line, type **1 EE (-) 13 ENTER**. It returns "1E-13". Now type **+ 1 ENTER**. It returns "1" because 1.0000000000001 has 14 digits so the last digit of 1 is hidden and rounds to 12 digits for display purposes only. To show you it's still there type **- 1 ENTER**. It returns "1E-13".

Now let's try this with $1\text{x}10^{-14}$. Type **1 EE (-) 14 ENTER**. It returns "1E-14". Type **+ 1 ENTER**. Like before, it returns "1". Again, type **- 1 ENTER**. But, now it returns "0". This happened because $1+1\text{x}10^{-14}=1.00000000000001$ which has 15 digits so to store it the calculator rounds to 14 digits, 1.0000000000000, losing the last digit of 1 instead of hiding it. Thus, when we subtracted 1, we subtracted it from 1 not 1.00000000000001. Which is why it returned "0".

So when $x > 2\text{x}10^{13}$

$1/x < 1/(2\text{x}10^{13})=0.5\text{x}10^{-13}=5\text{x}10^{-14}$

$1+1/x < 1.00000000000005$

which the calculator rounds to 1.0000000000000 . Therefore, when $x > 2\text{x}10^{13}$, the calculator rounds 1+1/x to 1. So when it then does $(1+1/x)^x$, it actually does $1^x=1$. This is why the calculator graphed a horizontal line from $2\text{x}10^{13}$ to the right.

When x is large but less than $2\text{x}10^{13}$ such as 30,000,001.

$1/x$	$= 1/30{,}000{,}001 = 3.33333322222\text{x}10^{-8}$	
$1+1/x$	$= 1.0000000333333$	rounding to 14 digits
	$= 1 + 333333/1\text{x}10^{13}$	from calculator $1\text{x}10^{13}/333333=30{,}000{,}030$
	$= 1 + 1/30{,}000{,}030$	so to the calculator $1+1/30{,}000{,}001=1+1/30{,}000{,}030$

$(1+1/x)^x = (1 + 1/30{,}000{,}030)^{30{,}000{,}001}$

so in effect the exponent is too small.

A strange partial fix.

Set y2=(1+1/x)^(1/(1/x+1-1)) , graph in the window of $[0,3\text{x}10^{13}]$ by [0,8] with a $\text{xscl}=1\text{x}10^{13}$ and go into trace mode. Remember, that y1 will graph also unless you deselect it.

Now, as long as $x<2\text{x}10^{13}$, this function approaches 2.7182818 which is e rounded to the 7th decimal place.

This works because in the exponent, adding 1 to 1/x then subtracting 1 forces the calculator to round it to the 13th decimal place. Then, doing the reciprocal of that rounded number, makes the exponent the value of x that was equivalently used in the base.

For example, when x=30,000,001

$1/x + 1$	$= 1 + 1/ 30{,}000{,}030$	(from above)
$1/x + 1 - 1$	$= 1/30{,}000{,}030$	
$1/(1/x + 1 - 1)$	$= 30{,}000{,}030$	
$(1+1/x)^{(1/(1/x+1-1))}$	$= (1+1/30{,}000{,}030)^{30{,}000{,}030}$	

which at least is in the form that I want. Thus, it gives a better result.

The problem with this method is we still have the $2\text{x}10^{13}$ limit. Unfortunately, there's no way around this limit. Even the most advanced of computers have a limit to the number of digits they store. Therefore, there would be some point at which 1+1/x would become 1.

Thus, the conclusion is:

YOU MUST BE SMARTER THAN THE CALCULATOR TO KNOW WHEN IT'S GIVING YOU GARBAGE!