

POLYnomial Solver

The polynomial solver can be used to solve polynomial equations of degree 2 or higher.

Example #1: Solve $x(x^2 - 3x) = 4$

1. Put the equation in standard form.
 $x^3 - 3x^2 - 4 = 0$
2. Enter the polynomial solver.
Type **2nd POLY**.
3. Enter the degree(order) of the polynomial.
Type **3 ENTER**.
4. Enter the coefficients starting with the leading coefficient.
Type **1 ENTER (-) 3 ENTER 0 ENTER (-) 4**. Note: you have to type in 0 for the missing x term's coefficient.
5. Press **F5:SOLVE** and wait.
6. Read the solutions. Note, if there are non-real solutions the calculator will display all solutions, even the real ones, in complex form. $a+bi = (a,b)$.

It returns $x_1 = (3.35530139761, 0)$
 $x_2 = (-.177650698804, 1.07730381285)$
 $x_3 = (-.177650698804, -1.07730381285)$

To see the rest of the second and third solutions, put the cursor on the solution and press (**cursor right**) to scroll through that solution.

Thus, rounded to the 4th decimal place, the solutions are
 3.3553
 -0.1777 + 1.0773i
 -0.1777 - 1.0773i

Note: This function is prone to approximation errors, i.e. 0's quite often show up as numbers like $3E-12=0.000000000003$. So if you get an answer like 1.9999999324, plug 2 in for x to see if the solution is 2 or 1.9999999324 . For example, try solving $x^5+5x^4+10x^3+10x^2+5x+1=0$ which has only one solution of $x=-1$ with multiplicity of 5.

SIMULTaneous Equations Solver

The Simultaneous equations solver is used to solve linear systems of square dimensions 2 by 2 or higher and have exactly one solution.

Example#4 :Solve $3x - 4y = 13$
 $x + 6y = 8$

1. Put all equations into general form, i.e. $Ax + By = C$.
2. Enter the Simultaneous equation solver.
Press **2nd SIMULT**.
3. Enter the number of equations or variables.
In this case type **2 ENTER**.
4. Enter the coefficients, in order, and the result for 1st equation.
Type **3 ENTER (-) 4 ENTER 13 ENTER**.
5. Repeat step #4 for the rest of the equations.
Type **1 ENTER 6 ENTER 8 ENTER**.
6. Press **F5:SOLVE**.
7. Read the answer.

It returns $x_1=5$, and $x_2=.5$ which means that $x=5$ and $y=0.5$. Thus the solution set is $\{(5, 1/2)\}$.

Equation editor's menu options:

- F1:PREV go to previous equation.
- F2:NEXT go to next equation.
- F3:CLRa clear all equations.
- F5:SOLVE solve the system.

Answer screen's menu options:

- F1:COEFS go back to equation editor.
- F2:STOa store the coefficients into a nxn matrix.
- F3:STOb store the results into a nx1 matrix(vector).
- F4:STOx store the solutions into a nx1 matrix.

Note: A Singular Matrix Error means that the system has either no solutions or infinite solutions. You have to determine which it is.

The TI-85 will sometimes return an answer when there are infinite solutions.

Equation SOLVER

The equation solver is used to find the value of any variable in an equation given all the other variables.

Storing the equation:

Example#2: Storing the Pythagorean theorem, $a^2 + b^2 = c^2$ into the calculator.

1. Solve for one of the variables.
I'll solve for c since this is the most unique of the variables. The other two are interchangeable. This gives $c = \sqrt{a^2 + b^2}$
2. Type in the equation using a more unique name for the variable you solved for. I'll use PYTH for c.
Type **ALPHA ALPHA P Y T H = 2nd ÷ ALPHA (ALPHA a x² + ALPHA b x²) ENTER**.
It will return "DONE".

Notes: The = sign is on the left 2nd from the bottom in blue.
The calculator is case sensitive, i.e. A#a.

Using the equation solver:

Example #3: Find the height of a right triangle whose base is 13ft. and whose hypotenuse is 15ft.

1. Enter the equation solver.
Type **2nd SOLVER**.
2. Since we've already stored the equation, clear the line.
Type **CLEAR**.
3. Find the equation's name in the menu, and enter it next to "eqn:".
At this point all of the equations in the calculator are listed in basically alphabetical order(all capital letters come before any lower case letter). If you don't see PYTH on the first page, then press **MORE** until you see it. When you find it, press the menu key(**F1 - F5**) under PYTH.
You should now have "eqn:PYTH" across the top of the screen.
4. Press **ENTER**.
5. You should now see all of the variables in the equation, except the equation's i.e. PYTH, listed down the left side of the screen. The second line "exp= " is used for the equation's variable, i.e. PYTH.
Ignore "bound={-1E99,1E99}" for most problems (see the error table below).
6. Type in the known variables.
In this case, c=PYTH=eqn=15 and b=B=13. Note: Actually A could be set to 13, but it's habit to label the base b.
Since the cursor should already be on eqn, type **15 ENTER**.
This will put the cursor on A and we need it on B, so type (**cursor down**) **13 ENTER**.
7. Put the cursor on the unknown variable.
At this point the cursor is on "bound={-1E99,1E99}". Thus type (**cursor up**) (**cursor up**).
8. Press **F5:SOLVE** and wait for an answer.
It returns "A=7.4833147735473", and "left-rt=0" appears at the bottom of the screen. The left-rt means that as far as the calculator is concerned there's no error in the value of A. But, keep in mind that the exact answer is $\sqrt{56}$, an irrational number. Therefore, the answer is accurate to the 13th decimal place, but it's still an approximation.
Thus, the base is 7.4833ft, rounded to the 4th decimal place.
9. To exit the solver, press **EXIT**.

Notes: The ▀ shows what lines were changed by the calculator.
Instead of looking for the equation's name in the menu, you can also type the name by "eqn:"
The values are stored on the calculator in the variables with the variable exp=15. So if you return to the equation solver with eqn=PYTH, the numbers will still be there unless you erased the variables or used them elsewhere.
You can erase the value of a variable by putting the cursor on that variable and pressing **CLEAR**.
If there is a number in the variable you are solving for, that number is used for the initial guess of the variable in the numerical process that the calculator uses to find the value of the variable. Otherwise, it averages the numbers in Bounds to get the guess.

Things that can cause errors:

ERROR	CAUSE	CURE
Variables don't show up in list.	Using a reserved variable.	Retype the equation changing the variable's name.
ERROR:Data type	A variable containing something other than a real number.	Erase the variable by putting the cursor on the variable and pressing CLEAR .
ERROR:No Sign Change, or ERROR:Iterations.	The process not converging to any number, or No real solutions.	If there are real solutions, enter your own guess for variable you're trying to find.
Converging to the wrong number.	More than one solution.	Enter your own guess for the variable you're trying to find or set bound to include the number it's returning and to include the number your trying to find. For example, if the calculator is returning "2" and you know the answer you want is between 5 and 6, set bound={5,6} .
ERROR:Bad Guess	Guess not in bounds.	Change the guess, or change the bounds.

TABLES (TI-86 ONLY)

The table feature on the TI-86 can be used to see a table of values for the function(s) in y_1 , and/or y_2 , etc.

Example#5 :Demonstrate that $(1 + 1/x)^x$ approaches $e=2.71828182845\dots$ as x approaches ∞ .

1. Enter the function into y_1 .
Press **GRAPH F1:y(x)= (1 + 1 ÷ x-var) ^ x-var**.
2. Enter the table menu.
Press **TABLE**.
3. Enter the table setup screen.
Press **F2:TBLST**.
4. Set TblStart to the value of x that you what the table to start at.
Set this to 1 by typing **1 ENTER**.
5. Set ΔTbl to how much you want x to change between rows of the table.
Set this to 1 by typing **1 ENTER**.
6. Leave Indpnt to AUTO.
7. Enter table screen.
Press **F1:TABLE**.
8. Examine the table.
Well the numbers under y_1 seem to be heading toward 2.7182818... , but they aren't really near it yet.
9. Experiment with the table setup.
Let's try $\Delta Tbl=1,000$. So type **F1:TBLST (cursor down) 1000 F1:TABLE**.
It looks better. At least it is accurate to the 3rd decimal place.
Let's try $\Delta Tbl=1,000,000$. So type **F1:TBLST (cursor down) 2nd (cursor right) 000 F1:TABLE**.
Much better. Now it's accurate to the 5th decimal place and the last entry, 2.718282, is what you get when you round e to the 6th decimal place.
From this we can conclude that $(1+1/x)^x$ seems to approach e as x approaches ∞ . But note that this is not proof that $(1+1/x)^x$ approaches e , see problem in next section. A formal proof requires calculus. But, this is enough for you to take my word that it does approach e .

When Good Calculators Go Bad

I came across this problem when working with the example used in the table section above (Example#5 :Demonstrate that $(1 + 1/x)^x$ approaches $e=2.71828182845\dots$ as x approaches ∞). We'll examine this both with tables and with graphs. So you with TI-85's just skip to the "With Graphs" subsection. Those with TI-86s should do both.

With Tables:

1. If it's not still there, set $y1=(1+1/x)^x$.
2. Set $\Delta Tbl=10,000,000$ and go to the table.
Does anything look a little peculiar? Look at when $x=30000001$.
3. Set $\Delta Tbl=3,000,000,000$ and go to the table. Note that you can use the EE key to type in ΔTbl a bit quicker.
Does it still look like it's converging to $e=2.7182818285\dots$?
4. Set $\Delta Tbl=1 \times 10^{13}$ and go to the table.
How about now? Are you questioning the conclusion we made in example#5?

With Graphs:

1. If not already done, set $y1=(1+1/x)^x$.
2. Set the window to $[0,10]$ by $[0,8]$ and graph.
Does it look like it could have a horizontal asymptote at $y=e=2.7182818285\dots$?
3. Set the window to $[0,1000000]$ by $[0,8]$ and go into trace mode.
Does it look like the function is approaching $2.7182818285\dots$?
4. Set the window to $[0,1 \times 10^{12}]$ by $[0,8]$.
How about now? Does it still look like it has a horizontal asymptote?
5. Try $[0,1 \times 10^{13}]$ by $[0,8]$.
Any ideas yet?
6. Finally, try $[0,5 \times 10^{13}]$ by $[0,8]$ and this time set $xScl$ to 1×10^{13} .
What happened at 2×10^{13} ?

The Limit of the Calculator.

First a couple of definitions. By a small number I mean a number close to 0, and by a large number I mean a number far from 0. Thus, 0.00000000005 is smaller than $-5,000,000,000$ even though 0.00000000005 is greater than $-5,000,000,000$.

The calculator can deal with numbers as small as 1×10^{-999} and as large as $9.999999999999999 \times 10^{999}$ with no problem as long as it doesn't try to mix large, small or average sized numbers at the same time. Then, there can be a problem depending on what you try to do with them. Multiplication is no problem, but addition is another story.

The next thing to understand is that the calculator stores at most 14 consecutive digits of any number. It displays the 1st 12 of those digits and keeps the other 2 hidden. To see this. On the command line, type **1 EE (-) 13 ENTER**. It returns "1E-13". Now type **+ 1 ENTER**. It returns "1" because 1.0000000000001 has 14 digits so the last digit of 1 is hidden and rounds to 12 digits for display purposes only. To show you it's still there type **- 1 ENTER**. It returns "1E-13".

Now let's try this with 1×10^{-14} . Type **1 EE (-) 14 ENTER**. It returns "1E-14". Type **+ 1 ENTER**. Like before, it returns "1". Again, type **- 1 ENTER**. But, now it returns "0". This happened because $1+1 \times 10^{-14}=1.00000000000001$ which has 15 digits so to store it the calculator rounds to 14 digits, 1.0000000000000 , losing the last digit of 1 instead of hiding it. Thus, when we subtracted 1, we subtracted it from 1 not 1.000000000000001 . Which is why it returned "0".

$$\begin{aligned} \text{So when } x &> 2 \times 10^{13} \\ 1/x &< 1/(2 \times 10^{13}) = 0.5 \times 10^{-13} = 5 \times 10^{-14} \\ 1+1/x &< 1.000000000000005 \end{aligned}$$

which the calculator rounds to 1.00000000000000 . Therefore, when $x > 2 \times 10^{13}$, the calculator rounds $1+1/x$ to 1. So when it then does $(1+1/x)^x$, it actually does $1^x=1$. This is why the calculator graphed a horizontal line from 2×10^{13} to the right.

When x is large but less than 2×10^{13} such as $30,000,001$.

$$\begin{aligned} 1/x &= 1/30,000,001 = 3.33333322222 \times 10^{-8} \\ 1+1/x &= 1.0000000333333 && \text{rounding to 14 digits} \\ &= 1 + 333333/1 \times 10^{13} && \text{from calculator} \\ &= 1 + 1/30,000,030 && \text{so to the calculator} \\ & && 1 \times 10^{13}/333333 = 30,000,030 \\ & && 1+1/30,000,001 = 1+1/30,000,030 \end{aligned}$$

$(1+1/x)^x = (1 + 1/30,000,030)^{30,000,001}$
so in effect the exponent is too small.

A strange partial fix.

Set $y_2 = (1 + 1/x)^{1/(1/x + 1 - 1)}$, graph in the window of $[0, 3 \times 10^{13}]$ by $[0, 8]$ with a $xScl = 1 \times 10^{13}$ and go into trace mode. Remember, that y_1 will graph also unless you unselect it.

Now, as long as $x < 2 \times 10^{13}$, this function approaches 2.7182818285 which is e rounded to the 10th decimal place.

This works because in the exponent, adding 1 to $1/x$ then subtracting 1 forces the calculator to round it to the 13th decimal place. Then, doing the reciprocal of that rounded number, makes the exponent the value of x that was equivalently used in the base.

For example, when $x = 30,000,001$

$$\begin{aligned} 1/x + 1 &= 1 + 1/30,000,030 && \text{(from above)} \\ 1/x + 1 - 1 &= 1/30,000,030 \\ 1/(1/x + 1 - 1) &= 30,000,030 \\ (1 + 1/x)^{1/(1/x + 1 - 1)} &= (1 + 1/30,000,030)^{30,000,030} \end{aligned}$$

which at least is in the form that I want. Thus, it gives a better result.

The problem with this method is we still have the 2×10^{13} limit. Unfortunately, there's no way around this limit. Even the most advanced of computers have a limit to the number of digits they store. Therefore, there would be some point at which $1 + 1/x$ would become 1.

Thus, the conclusion is:

YOU MUST BE SMARTER THAN THE CALCULATOR TO KNOW WHEN IT'S GIVING YOU GARBAGE!