

QUADRATIC FORMULA

If you use completing the square method on the general quadratic equation $ax^2 + bx + c = 0$ you will come up with the solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This is called the **quadratic formula**. It is a formula and you can sub in values of a , b , and c from the original equation into this formula, do some arithmetic and then you will have the solution set. This formula may be used to solve any quadratic equation, whether it factors or not. Please note that before you can properly identify a , b , and c the equation needs to be written so that 0 is on one side.

Let's try some examples.

Solve: $x^2 + 3x - 5 = 0$ Original equation with 0 on one side.

$$a = 1, b = 3, c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)} \quad \text{Sub in for } a, b, \text{ and } c.$$

$$x = \frac{-3 \pm \sqrt{9 + 20}}{2} \quad \text{Do the arithmetic and simplify.}$$

$$x = \frac{-3 \pm \sqrt{29}}{2} \quad \text{This is the solution set.}$$

Solve: $y^2 + 4y - 6 = 0$ Original equation with 0 on one side.

$$a = 1, b = 4, c = -6$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$

$$y = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-6)}}{2(1)} \quad \text{Sub in for } a, b, \text{ and } c.$$

$$y = \frac{-4 \pm \sqrt{16 + 24}}{2} = \frac{-4 \pm \sqrt{40}}{2} \quad \text{Do the arithmetic and simplify.}$$

$$y = \frac{-4 \pm 2\sqrt{10}}{2} \quad \sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$$

$$y = \frac{\cancel{2}(-2 \pm \sqrt{10})}{\cancel{2}} \quad \text{Factor out 2, then reduce.}$$

$$y = -2 \pm \sqrt{10} \quad \text{This is the solution set.}$$

Solve: $t^2 - 7t = 6$
 $t^2 - 7t - 6 = 0$
 $a = 1, b = -7, c = -6$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-6)}}{2(1)}$$

$$t = \frac{7 \pm \sqrt{49 + 24}}{2}$$

$$t = \frac{7 \pm \sqrt{73}}{2}$$

Original equation.
 Get 0 on one side of the equation.
 Quadratic Formula
 Sub in for $a, b,$ and $c.$
 Do the arithmetic and simplify.
 This is the solution set.

Solve: $u^2 - 2u + 3 = 0$
 $a = 1, b = -2, c = 3$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$u = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$u = \frac{2 \pm \sqrt{4 - 12}}{2} = \frac{2 \pm \sqrt{-8}}{2}$$

$$u = \frac{2 \pm 2i\sqrt{2}}{2}$$

$$u = \frac{\cancel{2}(1 \pm i\sqrt{2})}{\cancel{2}}$$

$$u = 1 \pm i\sqrt{2}$$

Original equation with 0 on one side.
 Quadratic Formula
 Sub in for $a, b,$ and $c.$
 Do the arithmetic and simplify.
 $\sqrt{-8} = \sqrt{-1 \cdot 4 \cdot 2} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2} = i \cdot 2\sqrt{2} = 2i\sqrt{2}$
 Factor out 2, then reduce.
 This is the solution set.

Solve: $5x^2 - x = 2$
 $5x^2 - x - 2 = 0$
 $a = 5, b = -1, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{1 \pm \sqrt{1 + 40}}{10}$$

$$x = \frac{1 \pm \sqrt{41}}{10}$$

Original equation.
 Get 0 on one side of the equation.
 Quadratic Formula
 Sub in for $a, b,$ and $c.$
 Do the arithmetic and simplify.
 This is the solution set.

Solve:

$$2u^2 = 6u + 3$$

$$2u^2 - 6u - 3 = 0$$

$$a = 2, b = -6, c = -3$$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$u = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-3)}}{2(2)}$$

$$u = \frac{6 \pm \sqrt{36 + 24}}{4} = \frac{6 \pm \sqrt{60}}{4}$$

$$u = \frac{6 \pm 2\sqrt{15}}{4}$$

$$u = \frac{\cancel{2}(3 \pm \sqrt{15})}{\cancel{2}}$$

$$u = \frac{3 \pm \sqrt{15}}{2}$$

Original equation.

Get 0 on one side of the equation.

Quadratic Formula

Sub in for a , b , and c .

Do the arithmetic and simplify.

$$\sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}$$

Factor out 2, then reduce.

This is the solution set.

Solve:

$$4x^2 = 16x - 28$$

$$4x^2 - 16x + 28 = 0$$

$$x^2 - 4x + 7 = 0$$

$$a = 1, b = -4, c = 7$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 28}}{2} = \frac{4 \pm \sqrt{-12}}{2}$$

$$x = \frac{4 \pm 2i\sqrt{3}}{2}$$

$$x = \frac{\cancel{2}(2 \pm i\sqrt{3})}{\cancel{2}}$$

$$x = 2 \pm i\sqrt{3}$$

Original equation

Get 0 on one side of the equation.

Divide both sides of the equation by 4.

Quadratic Formula

Sub in for a , b , and c .

Do the arithmetic and simplify.

$$\sqrt{-12} = \sqrt{-1 \cdot 4 \cdot 3} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{3} = i \cdot 2\sqrt{3} = 2i\sqrt{3}$$

Factor out 2, then reduce.

This is the solution set.

Solve: $(5w+2)(w-1) = 3w+1$

$$5w^2 - 3w - 2 = 3w + 1$$

$$5w^2 - 6w - 3 = 0$$

$$a = 5, b = -6, c = -3$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-3)}}{2(5)}$$

$$w = \frac{6 \pm \sqrt{36 + 60}}{10} = \frac{6 \pm \sqrt{96}}{10}$$

$$w = \frac{6 \pm 4\sqrt{6}}{10}$$

$$w = \frac{\cancel{2}(3 \pm 2\sqrt{6})}{\cancel{10}_5}$$

$$w = \frac{3 \pm 2\sqrt{6}}{5}$$

Original equation

FOIL

Get 0 on one side of the equation.

Quadratic Formula

Sub in for a , b , and c .

Do the arithmetic and simplify.

$$\sqrt{96} = \sqrt{16 \cdot 6} = 4\sqrt{6}$$

Factor out 2, then reduce.

This is the solution set.