EXPRESSIONS VERSUS EQUATIONS

Consider the following:

$\frac{1}{a-3}+\frac{2}{a+3}$	vs	$\frac{1}{a-3}+\frac{2}{a+3}=\frac{1}{a^2-9}$
This is an expression.		This is an equation.
This doesn't have an =		This does have an =
You simplify this		You solve this
Simplify by adding the fractions		Solve by finding out what a equals

Now let's look at each of these.

Simplify: $\frac{1}{a-3}+\frac{2}{a+3}$

You need a common denominator in order to add fractions. You are not getting rid of the fractions. The LCD is $(a-3)(a+3)$.

Convert each fraction to this denominator.

$\frac{1}{a-3}=\frac{1}{a-3}\cdot\frac{a+3}{a+3}=\frac{1(a+3)}{(a-3)(a+3)}=\frac{a+3}{(a-3)(a+3)}$ (Don't reduce or you will be going backwards.)

$\frac{2}{a+3}=\frac{2}{a+3}\cdot\frac{a-3}{a-3}=\frac{2(a-3)}{(a-3)(a+3)}=\frac{2a-6}{(a-3)(a+3)}$

Using these built up fractions with common denominators we can now add.

$$\frac{1}{a-3}+\frac{2}{a+3}$$

$$=\frac{a+3}{(a-3)(a+3)}+\frac{2a-6}{(a-3)(a+3)}$$

$$=\frac{a+3+2a-6}{(a-3)(a+3)}$$

$$=\frac{3a-3}{(a-3)(a+3)}$$

Always check to see if your answer will reduce. Even though this numerator will factor, it will not reduce with any of the factors in the denominator. Hence you are done. Again please note that the answer is also an expression. We have not solved for a since we did not start with an equation.

Now let's look at the equation.

Solve: $\dfrac{1}{a-3}+\dfrac{2}{a+3}=\dfrac{1}{a^2-9}$

Using a property of equality called the Multiplicative Property of Equality, we are allowed to multiply both sides of the equation by the same quantity. This will allow us to get rid of the fractions. You can do this with equations, not with expressions. We will multiply both sides by the LCD.

$$\frac{1}{a-3}+\frac{2}{a+3}=\frac{1}{a^2-9}$$

$$\frac{1}{a-3}+\frac{2}{a+3}=\frac{1}{(a+3)(a-3)}$$ The LCD is $(a+3)(a-3)$.

$$\frac{(a+3)(a-3)}{1}\cdot\frac{1}{a-3}+\frac{(a+3)(a-3)}{1}\cdot\frac{2}{a+3}=\frac{(a+3)(a-3)}{1}\cdot\frac{1}{(a+3)(a-3)}$$

$$\frac{(a+3)\cancel{(a-3)}}{1}\cdot\frac{1}{\cancel{a-3}}+\frac{\cancel{(a+3)}\,(a-3)}{1}\cdot\frac{2}{\cancel{a+3}}=\frac{\cancel{(a+3)}\,\cancel{(a-3)}}{1}\cdot\frac{1}{\cancel{(a+3)}\,\cancel{(a-3)}}$$

$$\begin{aligned}(a+3)\cdot 1+(a-3)\cdot 2&=1\\ a+3+2a-6&=1\\ 3a-3&=1\\ 3a&=4\\ a&=\frac{4}{3}\end{aligned}$$

Notice we end up with an answer for a. In that with equations you are solving for a value of the variable that satisfies the equation. Don't forget that these rational equations (equations with variables in the denominator) can have extraneous solutions. You must check to make sure that the answer you get does not cause any of the denominators to equal 0 in the original equation. Since $\frac{4}{3}$ does not make the denominators $a-3,\ a+3$ and $(a-3)(a+3)$ equal 0, it is a good solution.