COMPLEX NUMBERS

What happens when we try to take the square root of a negative number like $\sqrt{-4}$? We know this is not a real number because any real number multiplied times itself will be positive since the signs are alike. Well there are some more numbers beyond the real number system. This number set is called the set of **complex numbers**. It contains all of the real numbers we already know, but it also contains some numbers we have not studied yet. One of the numbers it contains that is not real is the number $\sqrt{-1}$. This number is used quite frequently and it is more convenient if we call it by another name. Its other name is i. i is called an **imaginary number**. Don't let its name fool you, it has lots of real-life applications.

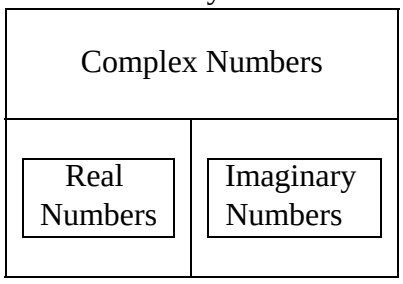
Definition: $i = \sqrt{-1}$ and $i^2 = -1$

We can use this to help us rewrite the nonreal number $\sqrt{-4}$.
$\sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{-1} \cdot \sqrt{4} = i \cdot 2 = 2i$
So $\sqrt{-4}$ is the same as $2i$.

$\sqrt{-36} = \sqrt{-1 \cdot 36} = \sqrt{-1} \cdot \sqrt{36} = i \cdot 6 = 6i$
So $\sqrt{-36}$ is the same as $6i$.

$\sqrt{-28} = \sqrt{-1 \cdot 4 \cdot 7} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{7} = i \cdot 2 \cdot \sqrt{7} = 2i\sqrt{7}$
So $\sqrt{-28}$ is the same as $2i\sqrt{7}$.

A **complex** number is a number that can be written in the form $a + bi$ where a and b are real numbers.
Some examples of complex numbers are $2 + 3i$, $7 - 8i$, $\frac{2}{3} - 4i$, $8 + i\sqrt{2}$
As we said before the real numbers are also in the complex number set. For example the real number 5 can be written as $5 + 0i$, hence it is a complex number. The imaginary numbers are the nonzero multiples of i. They also are in the complex number set. For example the imaginary number $2i$ can be written as $0 + 2i$, hence it is a complex number. Here is a diagram of our new expanded number system. The real numbers and imaginary numbers do not have any members in common, but they both are subsets of the complex number set.



We will be using complex numbers when we solve some quadratic equations. For now we just need to be able to simplify and write in terms of i.

Some more examples: Write using i notation.

1. $\sqrt{-49} = \sqrt{-1 \cdot 49} = \sqrt{-1} \cdot \sqrt{49} = i \cdot 7 = 7i$

2. $-\sqrt{-20} = -\sqrt{-1 \cdot 4 \cdot 5} = -\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{5} = -i \cdot 2 \cdot \sqrt{5} = -2i\sqrt{5}$

3. $4\sqrt{-18} = 4\sqrt{-1 \cdot 9 \cdot 2} = 4\sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{2} = 4i \cdot 3 \cdot \sqrt{2} = 4 \cdot 3i \cdot \sqrt{2} = 12i\sqrt{2}$