SOLVING RADICAL EQUATIONS

In order to solve a radical equation you must first get the radical term isolated by itself on one side of the equation. Then square both sides of the equation and you will eliminate the radical and thus be able to solve for x.

Example: Solve: $\sqrt{x-3}-1=0$

Isolate radical: Add 1 to both sides of the equation.

$$\sqrt{x-3}=1$$

Square both sides of the equation:

$$\left(\sqrt{x-3}\right)^2=1^2$$
$$x-3=1$$
$$x=4$$

We need to check this answer because radical equations can have extraneous solutions. Remember this means that even though you did everything correctly, the solution is not going to work in the original equation. To determine if this answer is a good solution simply plug it into the original equation and see if you get a true statement.

Original equation: $\sqrt{x-3}-1=0$

Plug in $x=4$: $\sqrt{4-3}-1=0$

$$\sqrt{1}-1=0$$
$$1-1=0$$
$$0=0$$

Since $0=0$ is a true statement, we know that $x=4$ is a good solution.

Let's try another example.

Original equation $\sqrt{2x-3}=3-x$

Square both sides $\left(\sqrt{2x-3}\right)^2=(3-x)^2$

$$2x-3=(3-x)(3-x)$$

FOIL $2x-3=9-6x+x^2$

Get 0 on one side $0=x^2-8x+12$

Factor $0=(x-6)(x-2)$

Set factors equal to 0 $x-6=0$ $x-2=0$

$$x=6 \qquad x=2$$

These two numbers must be checked in the original equation.

Original equation $\sqrt{2x-3}=3-x$

Sub in $x=6$ $\sqrt{2(6)-3}=3-6$

$$\sqrt{9}=-3$$

$3=-3$ False, hence $x=6$ is no good.

Original equation $\sqrt{2x-3}=3-x$

Sub in $x=2$ $\sqrt{2(2)-3}=3-2$

$$\sqrt{1}=1$$

$1=1$ True, hence $x=2$ is a solution.

The only solution to this equation is $x=2$.

Solving radical equations of other orders is very similar.

Solve: $\sqrt[3]{2x-6}-4=0$

Original equation	$\sqrt[3]{2x-6}-4=0$
Isolate the radical, add 4 to both sides	$\sqrt[3]{2x-6}=4$
Cube both sides to get rid of cube root	$\left(\sqrt[3]{2x-6}\right)^3=4^3$
	$2x-6=64$
Isolate x	$2x=70$
	$x=35$

You do not have to worry about extraneous solutions with odd-ordered radical equations, only even-ordered radical equations. However it is always a good habit to check your solutions.

Another example, this time with two radicals.

Solve: $\sqrt{3y+6}=\sqrt{7y-6}$

Original equation	$\sqrt{3y+6}=\sqrt{7y-6}$
Square both sides	$\left(\sqrt{3y+6}\right)^2=\left(\sqrt{7y-6}\right)^2$
Solve for y	$3y+6=7y-6$
Subtract $3y$	$6=4y-6$
Add 6	$12=4y$
Divide by 4	$3=y$

Don't forget to check this answer, it could be extraneous and you will not know until you sub it in and check it.

Check:

Original equation	$\sqrt{3y+6}=\sqrt{7y-6}$
Sub in 3 for y	$\sqrt{3(3)+6}=\sqrt{7(3)-6}$
	$\sqrt{15}=\sqrt{15}$ This is true, hence $y=3$ is a good solution.

Example: Solve: $\sqrt{x-2}+3=\sqrt{4x+1}$

Original equation $\qquad \sqrt{x-2}+3=\sqrt{4x+1}$

Be sure one of the radicals is isolated, then square both sides.

Square both sides	$\left(\sqrt{x-2}+3\right)^2=\left(\sqrt{4x+1}\right)^2$
FOIL	$\left(\sqrt{x-2}+3\right)\left(\sqrt{x-2}+3\right)=4x+1$
	$x-2+3\sqrt{x-2}+3\sqrt{x-2}+9=4x+1$
Combine like terms	$x+6\sqrt{x-2}+7=4x+1$

Isolate the remaining radical, you may keep its coefficient 6 with it.

Subtract x	$6\sqrt{x-2}+7=3x+1$
Subtract 7	$6\sqrt{x-2}=3x-6$
Square both sides	$\left(6\sqrt{x-2}\right)^2=(3x-6)^2$
FOIL on right side	$6^2\left(\sqrt{x-2}\right)^2=(3x-6)(3x-6)$
Simplify	$36(x-2)=9x^2-18x-18x+36$
Get 0 on one side	$36x-72=9x^2-36x+36$
Subtract $36x$	$-72=9x^2-72x+36$
Add 72	$0=9x^2-72x+108$
Divide all terms by 9	$0=x^2-8x+12$
Factor	$0=(x-6)(x-2)$
Set factors equal to 0	$x-6=0 \qquad x-2=0$
	$x=6 \qquad x=2$

Now these two values must be checked in the original equation.

Original equation $\sqrt{x-2}+3=\sqrt{4x+1}$

Sub in $x=6$ $\sqrt{6-2}+3=\sqrt{4(6)+1}$

$$\sqrt{4}+3=\sqrt{25}$$

$$2+3=5$$

$5=5$ True, hence $x=6$ is a solution.

Original equation $\sqrt{x-2}+3=\sqrt{4x+1}$

Sub in $x=2$ $\sqrt{2-2}+3=\sqrt{4(2)+1}$

$$\sqrt{0}+3=\sqrt{9}$$

$$0+3=3$$

$3=3$ True, hence $x=2$ is a solution.

Thus this equation has two good solutions and they are $x=6$ and $x=2$.

Final example, solve: $\sqrt{x+1}-\sqrt{x-1}=2$

Original equation $\sqrt{x+1}-\sqrt{x-1}=2$

Be sure one of the radicals is isolated, before squaring both sides.

Add $\sqrt{x-1}$ to both sides. $\sqrt{x+1}=2+\sqrt{x-1}$

Square both sides $\left(\sqrt{x+1}\right)^2=\left(2+\sqrt{x-1}\right)^2$

FOIL $x+1=\left(2+\sqrt{x-1}\right)\left(2+\sqrt{x-1}\right)$

$$x+1=4+2\sqrt{x-1}+2\sqrt{x-1}+x-1$$

Combine like terms $x+1=3+x+4\sqrt{x-1}$

Isolate the remaining radical, you may keep its coefficient 4 with it.

Subtract x $1=3+4\sqrt{x-1}$

Subtract 3 $-2=4\sqrt{x-1}$

Square both sides $(-2)^2=\left(4\sqrt{x-1}\right)^2$

$$4=4^2\left(\sqrt{x-1}\right)^2$$

Simplify $4=16(x-1)$

Distribute $4=16x-16$

Add 16 $20=16x$

Divide by 16 $\frac{20}{16}=x$

Reduce $\frac{5}{4}=x$

We must check this answer to make sure it is not extraneous.

Original equation	$\sqrt{x+1} - \sqrt{x-1} = 2$
Sub in $x = \frac{5}{4}$	$\sqrt{\frac{5}{4}+1} - \sqrt{\frac{5}{4}-1} = 2$

$$\sqrt{\frac{9}{4}} - \sqrt{\frac{1}{4}} = 2$$

$$\frac{\sqrt{9}}{\sqrt{4}} - \frac{\sqrt{1}}{\sqrt{4}} = 2$$

$$\frac{3}{2} - \frac{1}{2} = 2$$

$$1 = 2$$ This is false, hence $x = \frac{5}{4}$ is extraneous.

Since $x = \frac{5}{4}$ was our only possible solution and it is no good that means there is no solution to this equation. Or you can write $\emptyset$.